PSA 1994

Vienna Paper

* Insertion: The concept of local density is critically ~~examined~~ assessed.

The Vacuum in Relativistic Quantum Field Theory *

Michael Redhead.
University of Cambridge

1. Introduction
2. The Vacuum in non-relativistic quantum field theory
3. The relativistic Vacuum
   ~~Algebraic quantum field theory~~ - The Reeh
   ~~4 Nonlocality and the Vacuum~~ - Schlieder theorem
   Fluctuations and Correlations in the Vacuum
   ~~5 The Reeh-Schlieder Theorem and Vacuum~~ fluctuations.
   ~~6 The Bell inequalities and the Vacuum~~
   Nonlocality and the Vacuum.
   ~~7 The EPR argument and the Vacuum.~~
7-~~7~~8. Conclusion

Abstract The status of the vacuum ~~in relativistic~~ quantum field theory is examined. A sharp dependence arises between the global vacuum and the local vacuum *. The global vacuum state implies fluctuations for all local observables. Correlations between such fluctuations are ~~investigated~~ in space-like separated regions of space-time and discussed ~~investigated~~ and the existence of correlations which are maximal in a certain sense is ~~demonstrated~~ remarked on, independently of how far apart these regions may be. The analogy with the mirror-image

Insert
everywhere and at all times

correlations in the singlet state of two spin-½ particles as explained. The connection between these maximal correlations and the well-known violation of the Bell inequality in the vacuum is discussed together with the way in which the existence of these correlations might be exploited in developing a Vacuum version of the Einstein-Podolsky-Rosen argument. The recent relativistic formulation of the Einstein-Podolsky-Rosen argument by Ghirardi and Grassi is critically assessed with particular reference to the Vacuum case.

(174 words)

START THE MANUSCRIPT HERE, DIRECTLY AFTER HEADING

1. Introduction. [Intuitively the Vacuum state of the Universe is the state that would obtain if the spacetime around were emptied of its contents. The idea is that if we removed all the material particles, such as quarks and electrons together with all the force-carrying particles such as photons and gluons, so there was no matter or force in the Universe, then we would be left with a featureless spacetime, the bare arena of physics, and this would be the Vacuum.

There are many reasons why such an idea is problematic. On a strict relationist conception of spacetime, the procedure envisaged would leave not with empty spacetime but literally with nothing. Having removed the relata, the spatio-temporal relations would also disappear.

11

This problem does not arise for spacetimes of substantivalists, but general relativity raises new difficulties. Is there any sense in which we can talk of empty spacetimes of gravitation given the existence of non-trivial vacuum solutions of the Einstein field equations? The whole idea of an independent ground against which physics unfolds is of course expressly denied in general relativity. Another difficulty that arises is when we allow accelerated reference frames in the Unruh effect (1976). Here the detection of the vacuum or the absence of particles depends on the state of motion of the observer. So two mutually accelerating observers disagree on whether all the particles have been removed!

In this paper we shall ignore all these problems and deal only with special relativity and Minkowski spacetime. But already this will produce many surprises. We shall begin with the non-relativistic case, where the intuitive idea works very well and then show how things in relativity upset the intuitive approach.

2. De Vacuum in non-relativistic quantum field theory.

We choose briefly how to quantize the non-relativistic Schrödinger equation. For simplicity the field $\psi(\underline{x})$ and at a particular time $t=0$. Then the field $\psi(\underline{x})$ and its Hermitian conjugate $\psi^*(\underline{x})$ are interpreted as operators satisfying the equal-time commutation relations

$$[\psi(\underline{x}), \psi^*(\underline{x}')] = \delta(\underline{x} - \underline{x}') \quad \text{---(1)}$$

We introduce the number density operator

$$N(\underline{x}) = \psi^*(\underline{x})\psi(\underline{x}) \quad \text{---(2)}$$

To see why $N(\underline{x})$ is called a number density define $N_V = \int_V N(\underline{x})\, d^3\underline{x}$ ---(3)

Then it is easy to show that N_V has eigenvalues $0, 1, 2 \ldots$ the corresponding eigenstates are then interpreted as states in which there are $0, 1, 3 \ldots$ particles located within the spatial volume V.

However Moreover

$$[N_V, N_{V'}] = 0 \quad \text{---(4)}$$

for disjoint V and V'.

So we can simultaneously diagonalize N_V and $N_{V'}$, and in particular achieve a state in which both operators have the eigenvalue

the eigenvalues of

* given λ, N is the ~~total~~ relevant to total number of particles in field. But it is also to local Vacuum state

zero. We interpret this to mean that there are no particles in V or V'. In particular if V' is the complement of V in the whole of space we have the total number operator N as $N_V + N_{V'}$, and the eigenstate $|N=0\rangle$ is referred to as the vacuum state. We shall denote it by Ω. It is the global vacuum state in the sense that it is the zero eigenstate of N_V for all subvolumes V.

So, in a crisp slogan, the global vacuum implies a local vacuum. If there are no particles anywhere in space, then there are no particles present in any local region of space. This would like an analytic proposition. But amazingly it is just this nice meshing of the global and the local vacuua that breaks down in the relativistic case.

Another nice feature of the non-relativistic case is that everything works in an exactly parallel fashion in momentum-space as in position-space. Thus introducing Fourier transforms

$$\left.\begin{aligned} \psi(x) &= \frac{1}{(2\pi)^{3/2}} \int a(k)\, e^{(k\cdot x}\, d^3k \\ \psi^*(x) &= \frac{1}{(2\pi)^{3/2}} \int a^*(k)\, e^{-k\cdot x}\, d^3k \end{aligned}\right\} \quad (5)$$

then $a(k)$, $a^*(k')$ satisfy a commutation relation

$$[a(k), a^*(k')] = \delta(k - k') \quad \text{---} \quad (6)$$

which is exactly analogous to (1) and we can introduce a number density operator $N(\underline{k})$ in momentum-space defined as

$$N(\underline{k}) = a^*(\underline{k})\, a(\underline{k}) \qquad (7)$$

and it is easily shown that the total number operator N can be written equivalently as

$$N = \int N(\underline{k})\, d^3\underline{k} = \int N(\underline{x})\, d^3\underline{x} \qquad (8)$$

the first integral being extended over the whole of $\underline{k}$-space, and the second over the whole of $\underline{x}$-space.

Moreover we can introduce number operators over subregions of $\underline{k}$-space, exactly as we did in terms N_V, and check that the global vacuum is also a local vacuum in $\underline{k}$-space.

Curiously enough this feature is preserved in the relativistic case, where the treatments in $\underline{k}$-space and $\underline{x}$-space proceed quite differently, unlike the non-relativistic case we have been considering.

3. The relativistic vacuum.

It is easiest to start with the momentum-space treatment.

Consider as an example, the charged Klein-Gordon field at $t=0$.

The Fourier transform of the field $\phi(x)$ is now given by

$$\phi(x) = \frac{1}{\sqrt{2}} \cdot \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\omega(k)} \left[a(k)\, e^{ik\cdot x} + b^*(k)\, e^{-ik\cdot x} \right] \qquad (2)$$

where $\omega(k) = \sqrt{m^2 + |k|^2}$ and m is the mass parameter associated with the field. $\int \frac{d^3k}{\omega(k)}$ is an invariant measure in k-space.

Since ϕ is a scalar the operators $a(k)$ and $b^*(k)$ are also scalar functions of their arguments. Invariant commutator relations such as

$$\{ a(k), a^*(k') \} = \omega(k)\, \delta^3(k - k') \qquad (3)$$

are now enforced.

(3) should be compared with (6). Remember that in the non-relativistic limit $\omega(k)$ is just a constant m.

There is a similar commutator relation for b and b^*.

The total number of particles in the field is defined by

$$N^+ = \int \frac{d^3k}{\omega(k)}\, a^*(k)\, a(k) \qquad (4)$$

while the total number of anti-particles is defined by

$$N^- = \int \frac{d^3k}{\omega(k)}\, a^*(k)\, a(k) \qquad \text{——— (12)}$$

N^+ and N^- have the expected eigenvalue spectrum $0, 1, 2 \ldots$ and the global vacuum is the state $\Omega = |N^+ = 0, N^- = 0\rangle$. It is also a local vacuum in k-space, exactly as in the free relativistic case. But if we try to represent N^+ and N^- as spatial integrals over number density operators constructed out of scalar fields, call them $N^+(x)$ and $N^-(x)$ and construct spatial volume number operators such as

$$N^+_V = \int_V N^+(x)\, d^3x \qquad \text{——— (13)}$$

and similarly for N^-_V, then disaster strikes: in the sense that $[N^+_V, N^+_{V'}] \neq 0$ for disjoint V and V'. Similarly for N^-_V and $N^-_{V'}$.

So defining the total number of particles plus antiparticles by $N = N^+ + N^-$, then it follows that the global vacuum for which N has the eigenvalue 0, is not the vacuum state for $N^\pm_V$, for any sub-volume V.

So what has gone wrong? There are two/three typical responses in the literature:

(a) $N^\pm_V$ represents virtual particles and antiparticles whereas $N^\pm$ represent real particles.

So when there are no real particles present (in the vacuum) then what fluctuates locally is the number of virtual particles so preserving somehow the apparent paradox of saying that

when there no real particles present, the number of real particles is fluctuating locally. This approach is totally confused. Virtual particles arise in the context of interacting fields and have no place in the interpolation of free fields.

(2) It is possible to construct number density operators that do commute at space-like separation, if we give up manifest covariance of the description. Thus we can introduce so-called Newton-Wigner number densities $N^{\pm}_{n.w}(\underline{x})$ for particles and antiparticles and define

$$N^{\pm}_{V} = \int_{V} N^{\pm}_{n.w}(\underline{x})\, d^3x \qquad \text{--- (12)}$$

where $N^{\pm}_{n.w}(\underline{x}) = \psi^{\pm *}_{nw}(\underline{x})\ \psi^{\pm}_{nw}(\underline{x})$ — (13)

and
$$\psi^{+}_{nw}(\underline{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{w(\underline{k})}\, a(\underline{k})\, e^{(\underline{k}\cdot\underline{x})} \cdot \sqrt{w(\underline{k})}$$

$$\psi^{-}_{n.w}(\underline{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{w(\underline{k})}\, a^{\dagger}(\underline{k})\, e^{(\underline{k}\cdot\underline{x})} \cdot \sqrt{w(\underline{k})} \qquad \text{(14)}$$

It is now easy (check up - Eq. (10)) from the commutation relations for the a's and $a^{\dagger}$'s that

$$\left[\psi^{\pm}_{nw}(\underline{x}),\ \psi^{\mp}_{nw}(\underline{x}')\right] = \delta^3(\underline{x}-\underline{x}') \qquad \text{(15)}$$

From this it follows that

$$\{N^{\pm}_{\nu}, N^{\pm}_{\nu'}\} = 0$$

for any ν and ν', exactly as in the non-relativistic case.

It is easy now to interpret $2F^{\pm *}_{\nu\nu}(x)|0\rangle$ as states with a single particle or antiparticle located at the point x.

[However Newton-Wigner localizable [illegible] a number of unattractive features. Localizability is frame-dependent, for if we have a particle sharply at $x_n = 0$ in the rest-frame K, then in a moving-boost frame K', the particle would not be Newton-Wigner localizable at the origin.

It is easy to show that under a boost to the boosted frame the state

$$2F^{\pm *}_{\nu\nu}(\alpha)|0\rangle \text{ becomes.}$$

$$\frac{1}{(2\pi)^3}\int d^3x \, F^{\pm\alpha}_{\nu\nu}(x')\,F(x')\,|0\rangle\, d^3x'.$$

i.e. it is a superposition of states localized at various points x' along the spatial axis referred to the K'-frame.

The envelope factor $F(x')$ is given by

$$F(x') = \int d^3k \, e^{ik\cdot x'} \cdot \sqrt{\frac{w'(k)}{w(k)}} \qquad (5)$$

where $w'(k)$ is the boosted component of the [illegible]

If we had $w' = w = m$ as in the non-relativistic case, $F(x')$ would be just $\delta^3(x'-x)$, so the localized state would be localized at the origin. The non-invariance of the localized wave function comes from the factor $\sqrt{w(k)}$ in Eq. (14), which is also precisely what is needed to get the right commutator in Eq. (15). So Newton-Wigner localization is not Lorentz covariant, but it is a Lorentz frame. We can see that Newton-Wigner localized states are not well localized at all by computing the expectation value of the charge density $\rho(x)$ in the state that is Newton-Wigner localized at the point x'. The result does not vanish for $x \neq x'$ in spite of the charge on a Newton-Wigner localized state is not well localized, and also this difference vanishes in the large volume limit.

We turn therefore to a third approach to dealing with localized quantities in relativistic quantum field theory.

(c) We have seen how number density operators constructed in an invariant fashion fail to commute at space-like separations. Now it is usual in axiomatic formulations of quantum field theory to impose microcausality condition, or typically significant local observables, viz. that the associated operators should commute at space-like separation. The conclusion of this line of argument is that number density

Physical
are not observables, and hence we do not
have to worry about trying to interpret them.
There are lots of local observables such
as charge densities, energy densities and so on,
which do satisfy microcausality. The
global number operators $N_\pm$ are perfectly
respectable physical quantities which commute
with global quantities such as the total
charge: total energy etc, and thus the
case of charge for example, the total charge
Q commutes with the charge Q_V in a
sub-volume V, but it does not follow that
$N_\pm$ commutes with Q_V (a striking illustration
of the non-transitivity of commutativity!)
Indeed an eigenstate of $N_\pm$ acting on the vacuum state $|0\rangle$ does not have a
sharp value for Q_V, but exhibits quantum-mechanical
fluctuations! These vacuum fluctuations
of local observables are a characteristic feature
of the relativistic vacuum, and we shall
now proceed to investigate some remarkable
properties of these fluctuations in the
context of the axiomatic formulation of
algebraic quantum field theory.

4. Algebraic quantum field theory - The Reeh-Schlieder theorem.

The basic idea here is to associate a von Neumann algebra of local observables $R(O)$ with every bounded open region in space-time. In addition we assume a Hilbert space $\mathcal{H}$ in terms of which we can represent the action of a space-time translation a on the algebra $R(O)$ in the form

$$R(O+a) = U(a)\, R(O)\, U^*(a)$$

Here U is a unitary operator acting on $\mathcal{H}$ and $O+a$ is the image of O under the translation a.
For time-like translations $U(a)$ is exponentiated so as to obtain a Hamiltonian operator which is assumed to be non-negative, i.e. the energy spectrum of the field has no negative energies. In addition it is customary to introduce a global algebra R, defined as the smallest von Neumann algebra containing all the local algebras, and one assumes that the representation of R is irreducible and generated by the translates of $R(O)$ for any bounded open set O.
There are two important properties of the net of local algebras $\{R(O)\}$ which we shall assume:

Isotony: For any two bounded open sets O_1 and O_2, $O_1 \subseteq O_2 \Rightarrow R(O_1) \subseteq R(O_2)$

Microcausality: For all bounded open sets $\mathcal{O}_1$ and $\mathcal{O}_2$, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are space-like related (i.e. every point in $\mathcal{O}_1$ is space-like related to every point in $\mathcal{O}_2$) then every operator in $\mathcal{R}(\mathcal{O}_1)$ commutes with every operator in $\mathcal{R}(\mathcal{O}_2)$.

Finally we assume the existence of a global vacuum state Ω, defined to be the unique state which is invariant under every translation operator $U(a)$.

From these meagre postulates it is possible to derive one of the most famous results in axiomatic quantum field theory, the Reeh-Schlieder theorem (1961). This theorem states that Ω is a cyclic vector for $\mathcal{R}(\mathcal{O})$ with respect to the Hilbert space $\mathcal{H}$ for any $\mathcal{O}$. This just means that $\{A\Omega : A \in \mathcal{R}(\mathcal{O})\}$ is dense in $\mathcal{H}$, or in other words acting on Ω with operators in $\mathcal{R}(\mathcal{O})$ we can approximate as closely as we like any vector in $\mathcal{H}$. The theorem is remarkable because if $\mathcal{O}$ is for example, the neighbourhood of some particular point in space-time, how could $A\mathcal{R}(\mathcal{O})\Omega$ approximate an arbitrary state of the field, in particular one which looks quite unlike the vacuum in some distant, space-like separated neighbourhood $\mathcal{O}'$, without making a gross violation of locality? The explanation of this conundrum is that the vacuum is a highly correlated state and the Reeh-Schlieder theorem works essentially by exploiting the correlations between the local vacuum fluctuations in space-like separated regions.

In the next section I shall explain how

Therefore

$$\text{So } \Psi_{\text{singlet}} = \frac{1}{\sqrt{2}}\left(u_1 \otimes v_2 - v_1 \otimes u_2\right) \quad (16)$$

where u_1, v_1 are the eigenstates of the Pauli spin operator σ_{12} for particle one with eigenvalues $+1$ and -1 respectively. Similarly u_2, v_2 are spin states referring to particle two.

this works in the case of a greatly simplified model of space-time.

3 Fluctuations and Correlations in the Vacuum.

Imagine spacetime collapsed to a two points and suppose the Universe consists of just these two points with at each of which is located a spin-½ particle, and that the quantum state of the Universe is the famed entangled singlet state of the two particles, which we denote by $\Psi_{singlet}$.

Explain: So
$$\Psi_{singlet} = \frac{1}{\sqrt{2}}\left(|\sigma_{12}=+1\rangle \otimes |\sigma_{22}=-1\rangle - |\sigma_{12}=-1\rangle \otimes |\sigma_{22}=+1\rangle\right). \quad (16)$$

where σ_{12}, σ_{22} are the usual 2-components of the Pauli spin operators for the two particles. The Hilbert space for the Universe is just $|\Phi\rangle = \Psi_1 \otimes \Psi_2$ and each factor is associated with an algebra of spin operators R_1 and R_2.

Now in respect of spin, $\Psi_{singlet}$ is the analogue of the vacuum state Ω we had our explain. The local spin operators are all of the form $\sigma_1 \cdot \hat{n}$ and $(\sigma_2 \cdot \hat{n})$ for some direction $\hat{n}$ and the local projectors P_1, P_2 all project onto the $+1$ or -1 eigenvectors of $\sigma_1 \cdot \hat{n}$ $(\sigma_2 \cdot \hat{n})$ for some $\hat{n}$.

Now, from the rotational covariance of $\mathcal{H}_{singlet}$ we know that all the j spin operators have zero expectation value while for all the local projectors $\langle P_1 \rangle_{\mathcal{H}_{singlet}} = \langle P_2 \rangle_{\mathcal{H}_{singlet}} = 1/2$.

In other words, on the average there is no local manifestation of spin in any direction, but there is a probability of one half that the local spins will be found pointing along any given direction.

So the local spin components are entirely fluctuations with perfect mean-square correlation in any direction:

~~If we substitute charge for spin this is exactly~~ The situation is remarkably similar to the field theory vacuum.

In particular, it is easy to find a Fock version of the Bell-Schrödinger theorem that $\mathcal{H}_{singlet}$ is cyclic for R_1 (or R_2) with respect to $\mathcal{H}_{singlet}$. Full details can be found in Goddard (1995). Informally the proof exploits the correlations to manipulate the state in $\mathcal{H}_2$ by operations performed in $\mathcal{H}_1$ and vice versa, and hence show how an arbitrary state in $\mathcal{H}$ can be generated by operations performed in $\mathcal{H}_{singlet}$ which are elements of R_1 or R_2 ~~alone~~ only.

But can we turn the argument around and show from the Fock-Schrödinger theorem alone the existence of nonlocal correlations. The answer is yes, although what can be demonstrated is not all the features of $\mathcal{H}_{singlet}$, but just those which make the

proof of the Reeh-Schlieder theorem possible!
What we can show is the following

$$\forall P_2, \exists P_1 \text{ s.t. } \mathrm{Prob}^{\text{2f state}}(P_2 = 1 \,|\, P_1 = 1) = 1 \quad (17)$$

a equivalently

$$\forall P_2, \exists P_1 \text{ s.t. } \langle P_1, P_2 \rangle_{\text{2f state}} = \langle P_1 \rangle_{\text{2f state}} \quad (18)$$

the proof proceeds by considering the vector in H produced by property 2f state and P_2. Then, by the Reeh-Schlieder theorem can also be obtained by acting on a 2f state with some operator, and call it C_1, selected from the algebra of 2f state operators. Then it can be shown that the projectors onto the eigenstates of $C_1^* C_1$ satisfy the condition (18).

Of course, on the exact core of 2f state, the new set C_1 is itself just the mean-value projector, so $C_1^* C_1$ is just C_1 itself, and so itself the projector on P_1, where everything begins to degenerate!

But if we only go to the Reeh-Schlieder theorem then we have to use the more roundabout construction as introduced in Haag (1992).

But in the field theory case we know that the Reeh-Schlieder theorem holds, so we can prove something like (18), for any two spacelike related local algebras $R(\mathcal{O}_1)$ and $R(\mathcal{O}_2)$. The exact statement of the resulting theorem is

$$\forall \varepsilon > 0, \ \forall P_2 \in R(\mathcal{O}_2) \ \exists P_1 \in R(\mathcal{O}_1) \text{ s.t. } \langle P_1, P_2 \rangle_R > (1-\varepsilon) \langle P_1 \rangle_R \quad (19)$$

The assumption in (19) guarantees because cyclicity of Ω only requires that the set of vectors obtained by letting A act on Ω is dense in $\mathcal{H}$. Again reference may be made to Redhead (1980) for details of the rigorous proof of (19).

It is now to note carefully for the moment the Bell-Clauser-Horne-Shimony-Holt inequality (18) corresponds to the maximal value of the correlation coefficient $C(P_1, P_2)$ between P_1 and P_2 subject to fixed values of the marginal probabilities $\langle P_1 \rangle$ and $\langle P_2 \rangle$. It is only when $\langle P_1 \rangle = \langle P_2 \rangle = \frac{1}{2}$ that the maximal correlation attains the value 1. This condition is satisfied for Bohm's experiment, but it is not true in the Bell theory case. Indeed there is another famous theorem of axiomatic quantum field theory due to Frederhagen (1985) which provides a bound on $C(P_1, P_2)$ which falls off exponentially with the Bohm's minimum bound distance between θ_1 and θ_2.

Combining Frederhagen's theorem with (19) shows that for a fixed value of $\langle P_2 \rangle_\Omega$, i.e. of the probability of P_2 occurring in the vacuum state, then the maximally correlated P_1, whose existence is demanded in (19), must itself have a probability of occurring that falls off exponentially with the minimum bound distance between θ_1 and θ_2. The detection of P_1 may become effectively impossible if Bob's apparatus has its axis set at the scale of the separated beams, not

because P_1 fails to be maximally correlated with P_2, in the sense that $\mathrm{Prob}^{\Omega}(P_1=1 \,|\, P_2=1) = 1$, but because the marginal probability $\langle P_1 \rangle_\Omega$ became so small that the associated event hardly ever occurs!

As a final remark, let me state as a simple corollary of the Reeh–Schlieder theorem that Ω is also a separating vector for $\mathcal{R}(\mathcal{O})$ in the sense that

$$\forall A \in \mathcal{R}(\mathcal{O}),\ A\Omega = 0 \Rightarrow A = 0$$

(20)

This enables us to prove in a very simple and direct way that any local proposition P has non-vanishing expectation value in the vacuum, i.e. that there is a non-vanishing probability for the associated event to occur. Thus defining $p = \langle P \rangle_\Omega$, we have $p = \langle P^2 \rangle_\Omega = \|P\Omega\|^2$. So $p = 0 \Rightarrow P\Omega = 0$ $\Rightarrow P = 0$, ~~by the above~~ since Ω is a separating vector. Hence arguing contrapositively $P \neq 0 \Rightarrow p \neq 0$. This is illustrated in the Ising model case by merely noticing that all the local probabilities are equal to $\frac{1}{2}$ which is certainly not equal to zero!

So the picture we have of the vacuum is one in which any possible local event occurs with non-vanishing probability so all local propositions exhibit fluctuations. Vacuum fluctuations but at the same time these fluctuations are intricately correlated at every distant scale.

6. Nonlocality and the vacuum.

In the present section we want to discuss the question whether the vacuum correlations can be used to argue for nonlocal action-at-a-distance in the interpretation of relativistic quantum field theory, and hence to create a future tension with the requirements of relativity which led to the demonstration of the correlations in the first place! Let us rehearse briefly the situation in non-relativistic quantum mechanics taking again the familiar example of two spin-½ particles in the state Ψ^-. We assume either realism (R) namely that spin components have sharp values in Ψ^- or anti-realism (A) namely that they do not. Introducing two versions of locality

Loc_R: This principle prohibits the transfer from one sharp value to another sharp value effected by a local observable exhibited by some sort of space-like separation.

and Loc_A: This principle prohibits bringing a sharp value into existence at space-like separation

then the Einstein-Podolsky-Rosen argument (1935) asserts

$$A \Rightarrow D \wedge Loc_A \qquad (21)$$

while the Bell argument (1964), assuming the validity of the Bell inequality, asserts

$$R \Rightarrow D \wedge Loc_R \qquad (22)$$

whether we
do happen to realism or anti-realism in
~~interpretation~~ the interpretation of quantum
mechanics we are committed to some
form of nonlocality. For the sake of
argument we shall assume this conclusion to
be correct in the non-relativistic case.
We now want to see whether similar
conclusions hold in the case of the relativistic
domain.
Taking the case of the realistic interpretation
of the local hidden variables it is
well-known that the Bell argument can
be reproduced for the vacuum state.
So (22) still seems to hold. It is
worth remarking however, that even if
the Bell inequality is violated
so as to provide a common cause explanation of
the correlations by acts in the overlapping
backward light-cones of the correlated
events, this would still not give
an acceptable local explanation of
the correlations, this is because the vacuum
is time-translation invariant so any
common cause events appealed to explain
correlations at a later time, are themselves
inevitably included in ~~explicable~~ correlations
that themselves need explaining, so acts and
events on an infinite region of explanation,
or simply accept the correlations as
brute facts. So the use of nonlocality in
Bell inequality does not seem to succeed in
the vacuum state for giving either
acceptable causal explanations of the correlations
can be given.

Let us turn now to the anti-correlation option, and the problem of formulating the EPR argument in the vacuum state.

We recall briefly how the argument works in the non-relativistic case. Measuring the spin component in any direction for one of the space-like separated spin-½ particles enables us to predict the result of the anti-parallel measurement on the second particle, due to the perfect anti-correlations of the singlet. Applying the EPR criterion for identifying an element of reality allows us to infer the existence of a sharp value for the spin component of the second particle. Since at the anti-correlated orientation the sharp value did not exist prior to the measurement on the first particle, it must have come into existence by that measurement, a violation of LOCA, see (21) below.

The argument appears to fail in the relativistic case with measurements at space-like separation, where the concept of an invariant temporal order ceases to be unambiguous, and hence the EPR criterion in terms of possible precedence. However, the EPR criterion can also be given a counterfactual formulation, in a measurement were to be made on the second particle, then the calculated value would be found, assuming the existence of the element of reality in question. The counterfactual has been argued recently by Shimony and Stairs (1994) that the counterfactual criterion is still available in the relativistic case, and in particular we shall apply it to the

case of the maximal correlations in the vacuum described in Section 5. Thus we consider a measurement mode in a region O_2 that reveals the result one for the projector P_2 and ask the counterfactual question if we also measured P_1 in the space-like separated region O_1, that by (19) is maximally correlated with P_2, would the result also be one? If the answer is yes then according to the Ghirardi-Grassi criterion P_2 has a sharp value which is not the case in the vacuum state (unless that $1-P_1$ is also a projector in $\mathcal{R}(O_1)$ and since P_1 must have a non-vanishing probability of yielding the value zero in the vacuum as well as a non-zero value of yielding the value one). So we seem again to have demonstrated a violation of LOC_A. But here we have to be very careful. To derive the argument for violating LOC_A, we must ensure that if we did measure P_1, then would not affect the outcomes we do for the measurement of P_2. So we must invoke a principle which we call LOC_{M-O} which prohibits measurement outcomes being affected by other measurement procedures performed at space-like separation.

counterfactually

Then the full argument reads

$$A \wedge LOC_{M-O} \Rightarrow \neg LOC_A. \quad (23)$$

~~But now ask the question, is LOC_P~~

or

$$A \Rightarrow \neg LOC_{M-O} \vee \neg LOC_A \quad (24)$$

s/ we want to propose VOC_A then you can go to the left by going up VOC_{M-O}.
But now we can argue that modality VOC_{M-O} in a counterfactual understanding is perfectly acceptable and invokes no ~~unacceptable~~ special causal action, but just uses on a very natural reading of the truth conditions for counterfactuals in the presence of indeterminism. We argue as in ~~speak~~ the same as that taught by ~~Bedard (1987)~~ Vetter (1973) against the Stalk (1968) proof of the Bell inequality.

It then argued to accept that the derivation after does appear to ~~be~~ create ~~new~~ to particular argument, and hence ~~to~~ accept certain ~~quantum~~ bounded theory and spacetime relativity.

* ~~Acknowledgment~~. Grateful to David Robert for prompting me to clarify these issues in the first place.
~~— Also to Nancie Castle with whom I initially discussed this paper was completed in Idyllic Jamaica.~~

after 1st note

2 For details of the calculations see, for example, Henley and Thirring (1962, 44-45) who deal with the neutral Klein-Gordon field.

4 For full details of the algebra involved see Hannsky (1990)

5 For an overview of the controversies surrounding the conclusions see Redhead (1991)

Notes

1. By strict relationism here we are denying that relations between possible events could be constitutive of space-time.

2 ⇒

3. We also reserve Evrard & Parker - Wiper Cockburn to shortly responsible for the Jaynes regret of Hegfeldt (1974) & the subsequent threading of localized wave packets (on a given hand). See Redhead (1995) for a discussion of the relativistic Hegfeldt theorem and the Evrard - Parker Wiper Cockburn.

3 4 5. See Schrader and Wightman (1957, & Re. 831)

4 5. ⇒
5 6. ⇒

6 7. 4 See for example Landau ( ) and Summers and Werner (1988) and Landau (1987).

7 8 4. See for example Wendt (1981). For further discussion

8 9 6. This is also reserved by Shimony as Fraser, although they are not developed in great detail there. However, their argument for the condition is essentially based upon the one found here & A detailed critique of the Shimony - Fraser argument can be found in La Rivière and Redhead (1995).